

Performance Evaluation of Kinetic Code on Scalar Processors

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Eulerian Kinetic (Vlasov) Simulations for Space Plasma Studies

Basic equations for collisionless space plasma:

- Maxwell equations (for electromagnetic wave propagations)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$

Computational load less than 0.1%

- Collisionless Boltzmann equation with electromagnetic field (known as Vlasov equation for charged particle motions)

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

$$f(x, y, z, v_x, v_y, v_z)$$

6D! \Rightarrow 5D

Operator splitting into three equations [Umeda et al. 2012a]

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} = 0$$

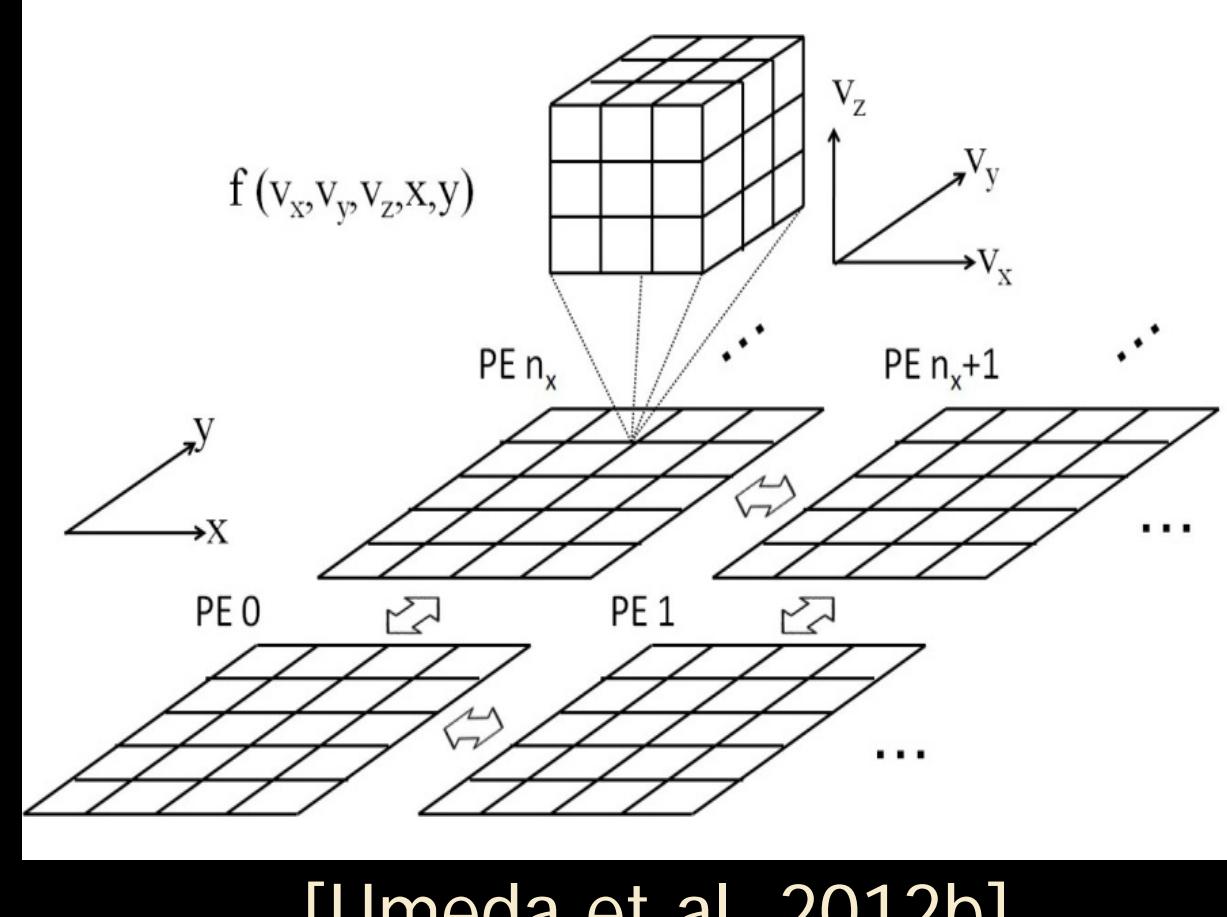
$$\frac{\partial f_s}{\partial t} + \frac{q_s}{m_s} \mathbf{E} \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

$$\frac{\partial f_s}{\partial t} + \frac{q_s}{m_s} (\mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

(advection in position by v) (advection in velocity by E) (rotation by B)

Non-oscillatory, positive and conservative interpolation is used

[Umeda 2009; Umeda et al. 2012]



$40^5 \sim 4\text{GB}$

$40^6 \sim 160\text{GB}$

- Hybrid parallelism is adopted to reduce number of processes
- Large number of dimensions (up to 6)
- \Rightarrow Requires huge memory
- Length of each loop is short: 20-40
- \Rightarrow number of threads > loop length in many core environments.
- Multiple loops are thread-parallelized by loop collapsing of OpenMP

[Umeda et al. 2016]

Performance Tuning

```

1 do n=0,nvx+1
2   do m=0,nvy+1
3     do l=0,nvx+1
4       ffi(l)=1.0d0/ff(l,m,n,i,j)
5     end do
6   end do
7
8   do l=0,nvx+1
9     gfyx = abs((gfy(l)*ffi(l))* gfy(l))           *0.5d0
10    gfyz = abs((gfy(l)*ffi(l))* g fz(l))           *0.5d0
11    gfzx = abs((gfz(l)*ffi(l))* g fx(l))           *0.5d0
12    gfxz = abs((gfx(l)*ffi(l))*(gfy(l)*ffi(l))*g fz(l))*inv3
13    dfx(l+10,m,n) = dfx(l+10,m,n) + (gfx(l)+(gfxz-gfx-y-gfx-z)*sx)
14    dfx(l+10,m,n+nv) = dfx(l+10,m,n+nv) - (gfyx-gfzx)*sx
15    dfx(l+10,m+mv,n) = dfx(l+10,m+mv,n) - (gfyx-gfxy)*sx
16    dfx(l+10,m+mv,n+nv) = dfx(l+10,m+mv,n+nv) + gfyx           *sx
17    dfy(l,m+m0,n) = dfy(l,m+m0,n) + (gfy(l)+(gfyx-gfyz-gfxy)*sy)
18    dfy(l+lv,m+m0,n) = dfy(l+lv,m+m0,n) - (gfyx-gfxy)*sy
19    dfy(l,m+m0,n+nv) = dfy(l,m+m0,n+nv) - (gfyx-gfyz)*sy
20    dfy(l+lv,m+m0,n+nv) = dfy(l+lv,m+m0,n+nv) + gfyx           *sy
21    dfz(l,m,n+n0) = dfz(l,m,n+n0) + (gfz(l)+(gfyx-gfzx-gfyz)*sz)
22    dfz(l,m+mv,n+n0) = dfz(l,m+mv,n+n0) - (gfyx-gfyz)*sz
23    dfz(l+lv,m,n+n0) = dfz(l+lv,m,n+n0) - (gfyx-gfzx)*sz
24    dfz(l+lv,m+mv,n+n0) = dfz(l+lv,m+mv,n+n0) + gfyx           *sz
25
26    end do
27  end do
28
29  do ll=nvx-1,nvx+1
30    ffi(l)=1.0d0/ff(ll,mm,nn,ii,jj)
31    gfx(ll) = abs(gfx(ll)*ffi)
32    gfy(ll) = abs(gfy(ll)*ffi)
33    gfz(ll) = abs(gfz(ll)*ffi)
34
35  end do
36  do ll=nvx-1,nvx+1
37    dfx(ll+10,mm,nn) = dfx(ll+10,mm,nn) + ff(ll,mm,nn,ii,jj)*gfx(ll)*(gfy(ll)*(gfz(ll)*inv3-0.5d0)+(1.0d0-gfz(ll)*0.5d0))*sx
38    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*(gfz(ll)*(gfx(ll)*inv3-0.5d0)+(1.0d0-gfx(ll)*0.5d0))*sy
39    dfz(ll,mm+mv,nn+n0) = dfz(ll,mm+mv,nn+n0) - ff(ll,mm,nn,ii,jj)*gfz(ll)*(gfy(ll)*(gfy(ll)*inv3-0.5d0)+(1.0d0-gfy(ll)*0.5d0))*sz
40
41  end do
42  do ll=nvx-1,nvx+1
43    dfx(ll+10,mm,nn+nv) = dfx(ll+10,mm,nn+nv) - ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*(gfz(ll)*inv3-0.5d0)*sx
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47  end do
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51    dfz(ll,mm+mv,nn+n0) = dfz(ll,mm+mv,nn+n0) + ff(ll,mm,nn,ii,jj)*gfz(ll)*gfy(ll)*(gfy(ll)*inv3-0.5d0)*sz
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53  end do
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