Numerical Linear Algebra Based on Lattice $\mathcal{H}$-Matrices

Akihiro Ida
The University of Tokyo
Bunkyo-ku, Tokyo,
Japan
ida@cc.u-tokyo.ac.jp

Ichitaro Yamazaki
Sandia National Laboratories
Albuquerque, New Mexico 87185
USA
iyamaza@sandia.gov

Rio Yokota
Tokyo Institute of Technology,
Meguro-ku, Tokyo,
Japan
rioyokota@gsic.titech.ac.jp

Satoshi Ohshima
Nagoya University
Chikusa-ku, Nagoya
Japan
ohshima@cc.nagoya-u.ac.jp

Tasuku Hiraishi
Kyoto University
Sakyo-ku, Kyoto
Japan
tasuku@media.kyoto-u.ac.jp

Takeshi Iwashita
Hokkaido University
Sapporo, Hokkaido
Japan
iwashita@iic.hokudai.ac.jp

Tetsuya Hoshino
The University of Tokyo
Bunkyo-ku, Tokyo,
Japan
hoshino@cc.u-tokyo.ac.jp

Toshihiro Hanawa
The University of Tokyo
Bunkyo-ku, Tokyo,
Japan
hanawa@cc.u-tokyo.ac.jp

ABSTRACT

In this research, we study algorithms for matrix arithmetic based on the lattice $\mathcal{H}$-matrix which is a new variant of low-rank structured matrices. In contrast to complicated structures of normal $\mathcal{H}$-matrices, lattice $\mathcal{H}$-matrices have the lattice structures which improve the convenience of matrix arithmetic and allow us to construct an efficient communication pattern on distributed memory computer systems.

KEYWORDS

matrix arithmetic, low-rank approximation, parallel computing.

To perform scientific simulations, we often need to deal with dense matrix operations such as matrix-vector and matrix-matrix products, matrix inversion, LU and QR factorizations. For the matrix size N, the dense matrix requires $O(N^2)$ memory, and the complexity of $O(N^3)$ for its factorization. To reduce the complexity, we can utilize an approximation method using low-rank structured matrices, such as BLR, HSS, HODLR and $\mathcal{H}$-matrices. Different low-rank structured matrices differ in partition structure and have different pros and cons in terms of the approximation accuracy, the compressibility and the convenience of matrix arithmetic. Our proposed lattice $\mathcal{H}$-matrices [1] can be expected to solve the trade-off problem. The lattice H-matrices appear to be a hybrid of $\mathcal{H}$- and BLR-matrices. Since lattice $\mathcal{H}$-matrices employ the strong admissibility condition, they can achieve high approximation accuracy with small number of rank. The memory complexity of lattice $\mathcal{H}$-matrices is $O(N \log N)$ which is the same as that of normal $\mathcal{H}$-matrices and less than $O(N^{1.5})$ of BLR-matrices. The lattice structures allow the use of existing algorithms for dense matrices because they look like a subdivided dense matrix. Moreover, thanks to the lattice structure, we can construct an efficient communication pattern among MPI processes.

By exploiting the advantages of lattice $\mathcal{H}$-matrices above, we are trying to establish numerical linear algebra to avoid the dense matrix operation (Fig. 1). Up to now, we achieve to develop algorithms for the matrix construction approximately discretizing an integral operator, the matrix-vector product and LU factorization [2]. These algorithms are implemented on distributed memory systems, and their performance are examined.

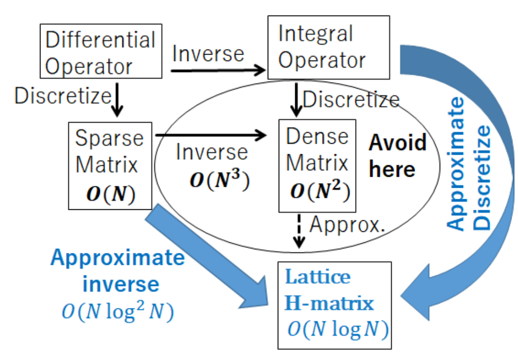

Figure 1: Conceptual image to avoid dense matrix operation using Lattice H-matrices.

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