A Periodic Table of Graphs with Special Properties

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The next-generation supercomputers reaching exascales require sophisticated and scalable interconnection networks to couple hundreds of millions of processing units and these interconnection networks must be scalable at high bandwidth and low latency. In addition to the requirement of many-host clusters, from the aspect of micro-chips, more and more cores are integrated on one chip that require a high-performance interconnection network to couple them. The processing speed of network-on-chip, an aggregated on-chip infrastructure to enhance the energy performance among others, sensitively relies on its framework design to maximize the latency reduction and throughput. Also, data-centers in cloud computing also demand optimal architectures for dealing with expeditious growth of nodes, memory, and interconnection networks in a supercomputing system.

It is obvious that most hierarchical networks are all based on Peterson graph and hypercube. Lacking diversity of orders, they limit the scales of clusters. Deng et al. [1] proposed more optimal graphs, with the benchmark results on a Beowulf cluster, which proves to enhance network performance than classical networks. Then, Xu et al. [2] applied these graphs to creating larger networks by using the Cartesian product, resolving scale limitations. To fill the family of optimal graphs, we use the exhausting method as mentioned in [3] to find the graphs with the minimal MPL, filtering with properties aiming to interconnection network, which can be used as NOC interconnect network directly, or combining with hierarchical method or Cartesian product to be used as the interconnect for clusters.

Criterions and optimal graphs: Our table, in a highly structured format, store all regular graphs with considering the following features,

- diameter and mean path length of regular graph
- bisection bandwidth
- automorphism group size

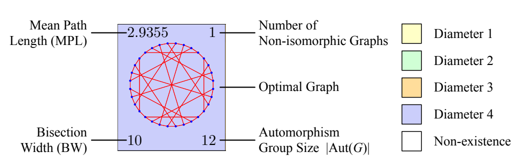
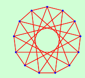
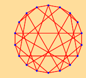
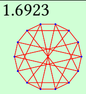
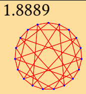
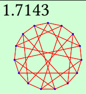
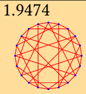
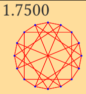
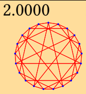
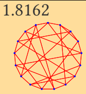
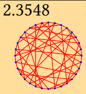


Figure 1: Graph legends.

Table 1: A subset of optimal base graphs of degree 4.

N \ k	4	N \ k	4
13	1.6667 1  10 52	18	1.8627 1  12 12
14	1.6923 1  10 96	19	1.8889 1  12 24
15	1.7143 1  10 12	20	1.9474 1  14 96
16	1.7500 1  12 32	21	2.0000 2  14 14
17	1.8162 1  12 36	32	2.3548 3  18 3

We searched for the optimal graphs first by enumeration using the same method as in [3] on supercomputers, among regular graphs of N number of vertices and degree k, denoted by (N, k) graphs. In particular, we exhaustively calculated the diameter and MPL of all $\sim 10^{12} - 10^{13}$ for $(21, 4)$ and $(32, 3)$ regular graphs. The optimal graphs, organized in Table 1, are a subset of all graphs listed in tables shown in [4], with colors indicating different minimal diameters and other parameters placed at the corners of the graphs as shown in the legend (Fig. 1). The asterisk marked on the parameter indicates not being equal to the theoretical lower bound, but still minimum.

Conclusions: Using the exhaustive methods, with supercomputers, we enumerate regular graphs with a wide range of orders, filter them with the diameter, MPL, symmetry, and bisection bandwidth. These optimal graphs can be applied to many areas from microchips to datacenters.

REFERENCES

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