IC(p) Preconditioning with Acceleration Factor for High-Frequency Electromagnetic Field Analysis

Koki MASUI
Osaka University
Japan
masui@ist.osaka-u.ac.jp

Masao Ogino
Daido University
Japan
m-ogino@daido-it.ac.jp

Takahiro KATAGIRI
Nagoya University
Japan
katagiri@cc.nagoya-u.ac.jp

Fumihiko INO
Osaka University
Japan
ino@ist.osaka-u.ac.jp

1. INTRODUCTION

Iterative methods suffer from oscillating residual norm histories and slow convergence rates in solving the complex symmetric linear equations derived from the edge finite element analysis of high-frequency electromagnetic fields [1]. Various iterative methods and preconditioners have been developed. Applying the preconditioning method such as Incomplete Cholesky (IC) preconditioning with acceleration factor (AF) is a common strategy to improve convergence in iterative methods for symmetric linear equations. Although IC(p) [2], where p is the fill-in level, can improve the convergence, there are few examples and an efficient implementation method has not been established in the past. Moreover, there is no example of applications for complex number problems such as electromagnetic field analysis. Therefore, we implement and optimize IC(p), and show the results when applied to electromagnetic field problems.

2. PRECONDITIONING

While IC(0) without fill-in generates the preconditioning matrix those non-zero element positions are the same as the coefficient matrix, IC(p) generates the preconditioning matrix with additional non-zero element positions depending on the value of p.

The additional non-zero element positions are determined by

$$Level_{ij} = \min\{Level_{ij}, \min_{1 \le k \le \min(i,j)} \{Level_{ik} + Level_{kj} + 1\}\}. \quad (1)$$

If we implement as per (1), the computational complexity will be $O(n^3)$, where n is the size of the matrix. Therefore, we implemented an efficient method that focuses on the position of non-zero elements in the coefficient matrix.

Figure 1 shows the method we used in this study. We compute additional non-zero element positions from the non-zero element positions contained in the column. If you want to compute IC(2), you should apply to the matrix computed by IC(1).

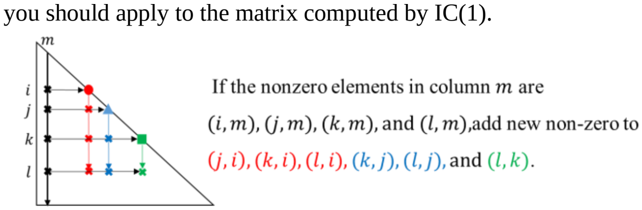

Figure 1. An efficient calculation method of preconditioning.

3. NUMERICAL EXPERIMENTS

As a high frequency electromagnetic field problem, let us consider a wave equation having an electric field derived from Maxwell's equation. We constructed a medium-scale problem with 134,573 degrees of freedom by ADVENTURE_Magnetic [3]. The number of nonzero elements in the coefficient matrix was 2,123,849. The number of non-zero elements in the preconditioning matrix in IC(0), IC(1), and IC(2) are 1,129,211, 2,094,069, 4,680,923, respectively.

All calculations were performed using an Intel CPU Core i9-9900 (3.60 GHz) processor with 32 GB memory, the gcc 10.2.0 compiler, and the "-O3 -mfma" optimization flag.

Figure 2 shows the iteration counts and calculation time as the AF of IC preconditioning was varied from 1.03 to 1.15 in increments of 0.01. As can be seen in the figure, increasing the fill-in level improved the convergence regardless of the acceleration factor, indicating that is effective in electromagnetic field analysis problems. Our proposed method succeeded in reducing the computation time by 51% compared to the conventional method (IC(0)) in the optimal parameters.

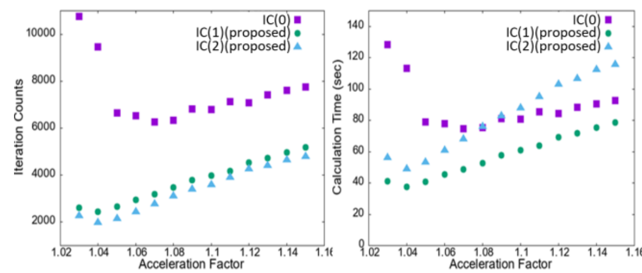

Figure 2. AF vs. iteration counts and calculation time.

4. CONCLUTION

In this study, we proposed an efficient IC(p) preconditioning method for complex symmetric matrices that appear in high-frequency electromagnetic field analysis and implemented an iterative method with different fill-in levels to evaluate the performance. As a result, our method succeeded in reducing both the number of iterations and the computation time and demonstrating its effectiveness in electromagnetic field analysis problems.

In future work, we aim to realize further acceleration by examining high-performance computing technology using Intel AVX instructions.

REFERENCES

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