Near kernel component setting method using iteration matrix in SA-AMG method

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1. INTRODUCTION

Analysis by computer simulation comes down to solving simultaneous equations $Ax=b$, and it is important to solve simultaneous equations fast and stably. One method for solving large-scale simultaneous linear equations is the SA-AMG method. This method coarsens the problem matrix in a hierarchical manner to obtain a solution, and is capable of solving large-scale problems at high speed. However, it depends on the problem setup. Conventionally, it is known that the convergence can be improved by setting the near kernel vector (0 eigenvalue component) [1]. In this study, we set the largest eigenvalue component of the transformation matrix applied to the error vector in the smoother as the component that is difficult to converge, and aim to verify its effectiveness.

2. CG method with SA-AMG preconditioning

The SA-AMG (Smoothed Aggregation Algebraic Multigrid) method generates a coarse lattice from an aggregate of the unknown variables in the fine lattice. The product of A and the interpolation operator generates each lattice hierarchically ($A_2 = R_2 A_1 P_2$).The coarse and fine lattices are solved alternately(The fine lattice attenuates the high-frequency components, while the coarse lattice attenuates the low-frequency components). Apply SGS (Symmetric Gauss-Seidel) smoothing at each level. Convergence can be improved by using components that are difficult to converge when generating interpolation operators.

3. Hard-to-Converge Components

Set a hard-to-converge component in the interpolation operator (R,P)used to generate each hierarchy. The hard-to-converge component is the largest eigenvector of the matrix $G(1)$ applied to the SGS error vector(D, L and U are diagonal, lower and upper part of the matrix A).

$$G = I - M^{-1}A = ((D+U)^{-1}L(D+L)^{-1}U) \qquad (1)$$

Using the power law, multiple maximum eigenvectors are simultaneously obtained and set as hard-to-converge components. Since this experiment uses the problem matrix of a three-dimensional structure, it has three unknowns per node. Therefore, to treat these unknowns together, the hard-to-converge components can be treated as a 3×3 block matrix Experiment.

4. Experiment

The problem matrix handled in the experiment is bone010.mtx (Oberwolfach:3D trabecular bone, Num Rows:986703, Nonzeros:47851783, Kind:Model Reduction Problem) obtained from the Suite Sparse Matrix Collection[2]. Measure solution convergence(Iterations) and run time (Setup Part, Iterative Part, NK Vec Part,NK=0 Part) when setting hard-to-converge components using matrix G(NK=0 : corresponding to using only 1 constant vector component). Compared results with blocking and without blocking for hard-to-converge components.

Figure 1 shows the results when the hard-to-converge components are set without using blocking. Figure 2 shows the results when the hard-to-converge components are set using blocking. The left vertical axis in Figures 1 and 2 is the run time, the right vertical axis is the number of iterations, and the horizontal axis is the number of hard-to-converge components.

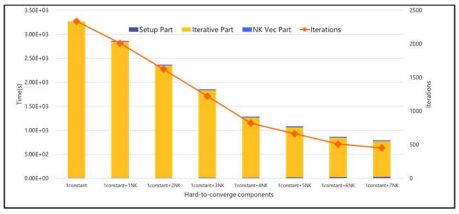

Figure 1. The hard-to-converge components are set without using blocking

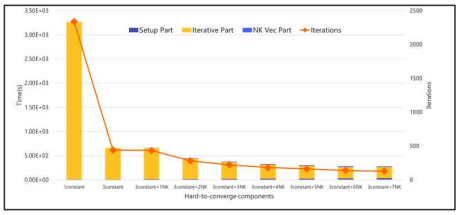

Figure 2. The hard-to-converge components are set using blocking

5. Conclusion

When the maximum eigenvalue component of the matrix G applied to the SGS (Symmetric Gauss-Seidel) error vector was set to the near kernel vector of the CG method with SA-AMG preprocessing, convergence was achieved in less than half as many iterations as those without the hard-to-converge component. Adding the hard-to-converge component using blocking was found to further improve convergence.

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